ANZATZ OF QUANTUM CORRELATION AND DISENTANGLEMENT PROBLEM

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Abstract

Pedagogical introduction into the problem of the mathematical description of the quantum correlation (entanglement) of composite quantum systems is represented. The notion is substantiated about the fact that the conventional algorithm of the reduction of von Neumann in the description of the dynamics of the observed subsystem is not universal and corresponds only to the case of maximum macroscopicity of the unobservable subsystem. Is clearly shown the sense of the algorithm of the correlated reduction proposed, which minimally changes the entropy of composite system.

1 Introduction. Theoretical difficulties in the description of quantum correlation.

The property of quantum correlation (or entanglement) of composite quantum systems became last decade one of the most discussed questions of quantum physics. The number of reviews and articles, in which is used the idea about the quantum correlation, continuously and almost exponentially grows according to the Gisin's estimation [1] since 1995. The reasons for this avalanche-type interest several, and even classification reasons appears ambiguous. Here we first attempt ourselves to trace how idea about the quantum correlation is expressed mathematically and, to what experimental operations correspond the accepted mathematical models . Primary attention is paid to the difficulties and the contradictions in the description and the interpretation of the experimental manifestations of quantum correlation. In the introductory part of the report we refer mainly to the last reviews, where it is possible to find the comprehensive bibliography.

The study of quantum correlation is conducted at present from different positions. Historically idea about the quantum correlation appeared with the examination of physical nature of quantum non-locality [1]. Another approach is connected with the description of the decogerence phenomenon of quantum systems due to the measurement and it is called frequently the problem of quantum measurements [2, 3]. The third, most "published" (according to the number of articles) approach, is connected with the development of quantum information theory and the examination of the possibility of using the quantum correlation for transfer and processing of information [4, 5]. In spite of the use of different terminology and interpretation, in all enumerated approaches it is easy to trace the united mathematical and physical content. Our first purpose, to show mathematical generality and physical content of differences and difficulties, which are in the different approaches.

Let us begin from the history, i.e., let us trace, what debating point, and what logical sequence of ideas and experiments led to the present situation.

In the work [1] this logical sequence of ideas is erected on the way of examining the axiomatically adopted (being seemed a priori obvious) property of the locality of nature -

interaction occurs three-dimensional locally, the transfer of interaction between the three-dimensional different points is accomplished by certain material agent, the velocity of propagation of which is limited. Newton clearly formulated the being contained in its theory of universal gravitation (but by them categorically rejected) property of the non-locality (stone moved on the Moon instantly changes the weight of any object on the Earth). Newton's non-locality mathematically is expressed in the absence in the law of universal gravitation of dependence on the time. To this, emphasized by Newton internal contradiction in the mathematical description of nature (mathematical model of gravity) the majority of physicists did not turn attention, since its experimental manifestations it was not observed. Non-locality (in the theory) was present up to 1915, when Einstein formulated general theory of relativity. Einstein by general theory of relativity returned to physics locality [1].

Quantum mechanics was borne in 10 years and again in the description of nature non-locality returned. Einstein in principle did not agree with the statistical interpretation of quantum mechanics, expressing the sense of disagreement by the known phrase:

> *God do not plays dice!*

Paraphrasing the statement of Einstein Gisin [1] formulates the fundamental results of the development of quantum physics of last decade by the assertion:

> *God does play dice, he even plays with nonlocal dice!*

Where in the quantum theory is contained quantum non-locality (quantum correlation, entanglement) and how it is expressed mathematically?

The unavoidable preamble of answer to this question is reminding the axioms of quantum mechanics and interpretation accepted, (see, for example, the recent account of this preamble in the thesis [4]). Let us limit this introduction, after isolating only those positions, around which now goes the discussion about the quantum correlation.

With the description of the quantum correlation:

1. One of the postulates of quantum theory is used - reduction postulate, i.e. the action of measurement consist in the collapse of wave function.

2. The tasks of quantum dynamics of composite quantum systems are examined.

Let the state of quantum system be represented by the wave function $|\Psi_l\rangle = \sum_\alpha (c_{l\alpha}|\alpha\rangle)$, $\sum_\alpha |c_{l\alpha}|^2 = 1$, where $|\alpha\rangle$ is the complete orthonormal set of the eigenstates (basis) of the considered system, or by the density operator $\hat{\rho} = \sum_l p_l|\Psi_l\rangle\langle\Psi_l|$, $\sum_l p_l = 1$, where the pure states $|\Psi_l\rangle$ are not compulsorily orthogonal. The set of eigenstates of the Hamiltonian of physical system is selected in many instances as the basis.

Let that observable $\hat{A}$ has eigenvalues a with the appropriate eigenvectors $|ad\rangle$, where the additional index d noted possible degeneracy $a = ad$. Then projector to the subspace of that observed $\hat{A}$ with the eigenvalue a is an operator $\hat{P}_a = \sum_{ad=a} |ad\rangle\langle ad|$. Mathematically the reduction postulate is expressed by the following conversion of clean wave function as a result of macroscopic measurement of observable a:

$$|\Psi_l\rangle \rightarrow |ad\rangle = \frac{\hat{P}_{ad}|\Psi_l\rangle}{\sqrt{\langle\Psi_l|\hat{P}_{ad}|\Psi_l\rangle}}. \tag{1}$$

Normalizing coefficient in the radicand of denominator is equal to the probability of the realization of this event. We will note that in the right side of (1) the equality is

If degeneration is absent, then this conversion is considered describing the ideal quantum von Neumann's measurement.

The conversion for the density operator corresponding to reduction is accepted to write in the form [4]

$$\hat{\rho} \to \hat{\rho}_{ad} = \frac{\hat{P}_{ad}\hat{\rho}\hat{P}_{ad}^{\dagger}}{\mathrm{Sp}\left(\hat{P}_{ad}\hat{\rho}\hat{P}_{ad}^{\dagger}\right)}, \tag{2}$$

with the assumptions of i) orthogonality $\mathrm{Sp}\left(\hat{P}_{ad}\hat{P}_{ag}^{\dagger}\right) = \delta_{dg}$ and ii) closure $\sum_{ad}\hat{P}_{ad}\hat{P}_{ad}^{\dagger} = \hat{I}$, where $\hat{I}$ is the unit operator and subscript N here and subsequently will be used for the designation of that reduction according to the axiomatic von Neumann's algorithm. Normalizing factor in denominator of (2) it is possible to copy in the different form, since the trace does not depend on the non-diagonal matrix elements of operator. Conversions (1),(2) relate to the arbitrary closed quantum system, on which is conducted the measurement by external macroscopic measuring device. It is necessary to emphasize that very start of measurement is passage to the open system, but all the examination is conducted in the Hilbert space of the closed system.

The property of quantum correlation is determined for the composite quantum systems. Generally speaking, in quantum physics the composite systems are always examined, since the reduction postulate, being the inherent part of the quantum theory, implies existence of two physically divided parts: the described quantum system and measuring (macroscopic) device. However, usually, keeping in mind composite quantum system, it is assumed that they are determined (i.e. experimentally they can be independently isolated and studied) several quantum subsystems $A, B, E, \ldots$(in the quantum informatics corresponding terms are accepted - Alice, Bob, Eve-Eavesdropper, Charlie...). The certainty of such subsystems means that are considered known their Hamiltonians $\hat{H}_A, \hat{H}_B, \hat{H}_E, \ldots$, which correspond eigenstates $|\alpha\rangle, |\beta\rangle, |\varepsilon\rangle, \ldots$ (frequently selected as the bases), the operators of those observable $\hat{A}, \hat{B}, \hat{E}, \ldots$, their eigenvalues $a, b, e, \ldots$ and eigenstates $|a\rangle, |b\rangle, |e\rangle, \ldots$. We for the simplification not will here introduce special indices for the designation of possible degeneracy. Furthermore, are considered known the Hamiltonians of pairwise interactions between the subsystems $\hat{H}_{AB}, \hat{H}_{AE}, \hat{H}_{BE}, \ldots$, moreover interactions are relied by the relatively weak $\left(\hat{H}_{AB}, \hat{H}_{AB}, \ldots\right) < \left(\hat{H}_A, \hat{H}_B, \ldots\right)$ (there are in the form the relative values of the eigenvalues of energy). The obvious terminology is accepted: bipartite system $\left(\hat{H}_{A+B}\right)$, tripartite system $\left(\hat{H}_{A+B+E}\right), \ldots$ multipartite system $\left(\hat{H}_{A+B+E+\ldots}\right)$.

The majorities of the fundamental properties of quantum correlation can be examined based on the example of the bipartite quantum system $\left(\hat{H}_{A+B}\right)$. The relative smallness of interactions between the subsystems makes possible to use the first mathematical assumption - to describe the state of composite system in the Hilbert space of dimensionality $N_A \times N_B$, which is been the direct product space of the space of the independent subsystems , where N_A, N_B are the dimensionality (number of eigenstates) of subsystems respectively. I.e. the set of pared multiplications of eigenvectors of subsystems $|\alpha\rangle \times |\beta\rangle$ is selected as basis. Let the state of system in this basis be described by the density operator of $\hat{\rho}_{A+B}$ $(\hat{\rho}_{A+B} \in \hat{H}_A \otimes \hat{H}_B)$.

There is one additional taciturn adopted (implicit) limitation to many composite quantum systems, for which is determined the property of quantum correlation. It is assumed that there is a physical possibility to conduct local actions or measurements on the subsystems [6]. Term "local" here does not bear in the general case of the content "three dimensional localization", although in many experimental cases this precisely thus

Mathematically this possibility is described by assumption about the validity of existence of the following (local) maps of the states of the quantum system:

$$\hat{\rho}'_{A+B} = \left(\hat{U}_A \otimes \hat{U}_B\right)\hat{\rho}_{A+B}\left(\hat{U}_A \otimes \hat{U}_B\right)^{\dagger},\tag{3}$$

where $\hat{U}_A, \hat{U}_B$ are unitary operators, determined in the spaces of the subsystems, by which is described the action (most frequently external) on the subsystems. The equality $\hat{U}_{A(B)} = \hat{I}_{A(B)}$, where $\hat{U}_A$ are unit operators in the space of the corresponding subsystem, corresponds to the absence of local action on the subsystem.

Existence of quantum correlation (entanglement) between the subsystems of such composite system is accepted to mathematically determine through the opposite property of the inseparability of quantum system, i.e., the impossibility to represent the density operator in the form of the convex linear combination of the direct (tensor) products of the pure density operators of subsystems [7]:

$$\hat{\rho}_{(A+B)s} = \sum_l p_l\left(\hat{\rho}_{Al} \otimes \hat{\rho}_{Bl}\right),\tag{4}$$

where $0 < p_l \leq 1$, $\sum_l p_l = 1$, $(\hat{\rho}_{Al})^2 = \hat{\rho}_{Al}$, $(\hat{\rho}_{Bl})^2 = \hat{\rho}_{Bl}$, $\hat{\rho}_{Al}$, $\hat{\rho}_{Bl}$ are the density operators of subsystems. With the validity of equality (4) the system is considered separable that marked here by additional subscript s. In the separable system quantum correlation (entanglement, quantum non-locality) is absent. In this case statistical weights p_l it is accepted to call the hidden parameters of quantum system. The states of the composite quantum system of form (4) can be obtained by local operations (3) from the originally separated subsystems.

Answer to the question: is the state (the density operator) of composite quantum system separable, or entangled? - it is accepted to call the separability or disentanglement problem. At present the mathematical disentanglement problem seems unsolvable in the general case for two reasons [4]. From one side there is an infinite set of expansions of the separable density operator of form (4), since the number of clean and not compulsorily orthonormal states of subsystems is unconfined by anything. Physically to this the infinite set of the ways of creating this separable state with the aid of the local operations corresponds. From other side there is an intriguing mathematical property: the linear combination of the entangled states can be the separable state. But at the same time the linear combination of the separable states is always the separable state.

In what the physical sense of quantum correlation and why must be known, are entangled subsystems or not? The traditional setting of physical experiments consists of the observation of the dynamics of physical subsystems as a result of their interaction. In many cases the state of composite system, described by the density operator, is obtained as a result of solving (most frequently the approximate) dynamic Schroedinger (Neumann) equation as the result of certain interaction between originally separated subsystems. The mixed character of this state (i.e. its description by the density matrix, but not by wave function) can appear, only if there are some uncontrollable degrees of freedom out of the subsystems in question. If state is described by the entangled density matrix, then between the subsystems there is (or there existed and it left its track) certain coherent interaction. But if the density matrix is separable, then was unknown interaction between the subsystems coherent, or not, and did remain there what or tracks of the mutual coherence of subsystems. Here term coherence is identical to term "quantum coherence", i.e., the importance of the phase relationships between the subsystems in the process of interaction and up to the moment of measurement.

Attempts at the mathematical solution of the disentanglement (separability) problem

unresolved problem of the characterization of positive maps corresponds) [4],[6],[5]. All approaches use as the criterion of separability disturbance of one of the physical limitations to the characteristics of the density operator. Let us group the mathematical criteria of the separability (entanglement) proposed in their physical content.

1. **Positive partial transpose (PPT) criterion.** This is necessary, but insufficient criterion of separability. In order to clearly present its content let us write down the density operator $\hat{\rho}_{A+B}$ in the selected basis:

$$\hat{\rho}_{(A+B)} = \sum_{\alpha,\alpha'} \sum_{\beta,\beta'} \langle\alpha\beta|\hat{\rho}_{(A+B)}|\alpha'\beta'\rangle |\alpha\rangle_A \langle\alpha'| \otimes |\beta\rangle_B \langle\beta'|. \tag{5}$$

Then the density operator partially transposed on the subsystem A is determined by the expression:

$$\left(\hat{\rho}_{(A+B)}\right)_{T(A)} = \sum_{\alpha,\alpha'} \sum_{\beta,\beta'} \langle\alpha\beta|\hat{\rho}_{(A+B)}|\alpha'\beta'\rangle |\alpha'\rangle_A \langle\alpha| \otimes |\beta\rangle_B \langle\beta'|. \tag{6}$$

System is separable, if this operator $\left(\hat{\rho}_{(A+B)}\right)_{T(A)}$ has only positive eigenvalues. (It is equivalent for the transposition on the subsystem B: $\left([\hat{\rho}_{(A+B)}\right)_{T(B)}$, moreover $\left(\left(\hat{\rho}_{(A+B)}\right)_{T(A)}\right)_{T(B)} = \left(\left(\hat{\rho}_{(A+B)}\right)_{T(B)}\right)_{T(A)} = \left(\hat{\rho}_{(A+B)}\right)_T$). The condition of positive partial transposition makes simple physical sense. The separable system satisfies condition (4), which shows that the dynamics of subsystems occurs it independently and, therefore, satisfies the condition of local unitarity (3), i.e., reversibility in the time. The operation of transposition on the subsystem corresponds to time reversal in this subsystem. Consequently, the condition of positivity corresponds to the absence of nonphysical (negative) probabilities with the time reversal in one of the subsystems.

2. **Reduction criterion.** In the mathematical algorithm the reduction criterion is close to PPT criterion, since the conversion on one of the subsystems there also is done. It is possible to formulate this criterion as follows: system is in the separable state, if inequalities simultaneously are fulfilled [4]:

$$\frac{1}{N_A}\hat{I}_A \otimes \hat{\rho}_{BN} - \hat{\rho}_{(A+B)} \geq 0, \ \hat{\rho}_{AN} \otimes \frac{1}{N_B}\hat{I}_B - \hat{\rho}_{(A+B)} \geq 0, \tag{7}$$

where $\hat{\rho}_{AN} = \mathrm{Sp}_B\hat{\rho}_{(A+B)}$, $\hat{\rho}_{BN} = \mathrm{Sp}_A\hat{\rho}_{(A+B)}$ are reduced density operators of subsystems. Inequalities for the eigenvalues of operators here are implied, i.e. is assumed passage to the basis, in which the operators are diagonal.

The enumerated criteria carry to the operational, i.e., to such, for which the algorithm of calculation and relationship for some values, which determine the conditions of separability, is indicated. The physical sense of last criterion is different from that indicated in the first point. It is possible to connect it with distinctions in kind in the entropy of the entangled and separeble composite quantum systems. Therefore let us transfer the first known properties of the entropy of composite quantum systems. Very determination of entropy is in this case debating point with the ambiguous and sometimes contradictory terminology.

The quantity of information is evaluated , which we know about the quantum sys-

First remind of the von Neumann's determination of the entropy of the quantum system, described by the density operator $\hat{\rho}$:

$$S\left(\hat{\rho}\right) = -\mathrm{Sp}\left(\hat{\rho}\ln\hat{\rho}\right). \tag{8}$$

It is assumed that with the calculation of this value they pass to the basis, in which the density matrix is diagonal. Entropy describes the deflection of quantum system from the pure state. Its property:

- Entropy is equal to zero (it is minimum!) for the pure state. Entropy is maximum and equal to $\ln N$ for a maximally mixed state, when $\hat{\rho} = (1/N)\hat{I}$. In the first case is known maximally possible, and in the second - minimally possible information about the quantum system [8]. Let us note the opposition of terminology from the accepted in the quantum informatics definition of the characteristics of the states of quantum system from the possibility of the content in them of minimum ($S(\hat{\rho}) = 0$) and maximum ($S(\hat{\rho}) = \max$) quantity of information [9]. Entropy is invariant with the unitary conversions of the basis: $S(\hat{\rho}) = S\left(\hat{U}\hat{\rho}\hat{U}^{\dagger}\right)$.

- The entropy of the separable quantum systems is additive, i.e. $S\left(\hat{\rho}_A \otimes \hat{\rho}_B\right) = S(\hat{\rho}_A) + S(\hat{\rho}_B)$. However, for the composite entangled quantum system is characteristic the property of sub-additivity, determined by the inequality: $|S(\hat{\rho}_{AN}) - S(\hat{\rho}_{BN})| \leq S(\hat{\rho}_{A+B}) \leq S(\hat{\rho}_{AN}) + S(\hat{\rho}_{BN})$. According to Shannon's theory, the entropy of composite system never can be less than the entropy of any of its parts. For the entangled system with the local reduced (according to von Neumann's algorithm) density operators - this is incorrect. It is considered that this property of entropy can serve as the indicator of the entanglement of state. However, there is a separate direction in quantum informatics, which argues the inapplicability of the classical determination of information to the composite quantum systems and is proposed a number of the alternative determinations of entropy for the composite quantum systems for the purpose to return by it the property of additivity [4].

It is considered that existence of entanglement experimentally is manifested in the inequality $\langle\hat{A}\hat{B}\rangle \neq \langle\hat{A}\rangle_{\mathrm{N}}\langle\hat{B}\rangle_{\mathrm{N}}$. To the left stands the experimentally specific average value of that nonlocal observable, specific by expression $\langle\hat{A}\hat{B}\rangle = \mathrm{Sp}\hat{\rho}_{A+B}\left(\hat{A} \otimes \hat{B}\right)$. The product of the calculated local average values, determined by equations $\langle\hat{A}\rangle_{\mathrm{N}} = \mathrm{Sp}\hat{\rho}_{A+B}\hat{A}'$, $\langle\hat{B}\rangle_{\mathrm{N}} = \mathrm{Sp}\hat{\rho}_{A+B}\hat{B}'$ to the right stands. The upper prime noted operators, extended to the complete space: $\hat{A}' = \hat{A} \otimes \hat{I}_B, \hat{P}'_B(\beta) = \hat{I}_A \otimes \hat{P}_B(\beta)$. It is accepted to name the specially selected nonlocal observable $\hat{W}$ for the concrete systems the witness of entanglement. It is proven that the composite bipartite system is entangled if and only if there is witness of entanglement - Hermitian operator $\hat{W}$ ($\hat{W} = \hat{W}^{\dagger}$), for whom are valid the inequalities: $\mathrm{Sp}\hat{\rho}_{A+B}\hat{W} \leq 0$ while $\mathrm{Sp}\hat{\rho}_{(A+B)s}\hat{W} \geq 0$ for all separable states $\hat{\rho}_{(A+B)s}$. This is the sufficiently abstract non-operational criterion of quantum correlation, to which it is difficult to compare literal physical sense, however precisely this form of the inequality (them proposed much for the different experimental situations) is taken asd the experimental test of the quantum correlation of subsystems.

Let us emphasize that it is always assumed that with the variety of approaches to the description of quantum correlation the local operators of the density of subsystems (and, correspondingly, the local dynamics of subsystems) are determined by the reduction algorithm of von Neumann ($\hat{\rho}_{AN}(t) = \mathrm{Sp}_B\hat{\rho}_{A+B}(t)$, $\hat{\rho}_{BN}(t) = \mathrm{Sp}_A\hat{\rho}_{A+B}(t)$). Our work is

algorithm of the determination of the local density operators, its injustice in the general case and the needs for the calculation of the mutual correlation of subsystems with the description of their local dynamics.

The idea of examination is based on what non-universality of von Neumann's algorithm is already repeatedly demonstrated in different physical tasks. Apparently, historically as this first example can serve the construction of the quantum theory of relaxation [8],[10]. There one of the interacting subsystems (let B) from the physical considerations is relied stationary, that possesses by the properties of quasi-classical thermostat. Its state is approximately described by the Boltzmann's density operator with the specific temperature T: $\hat{\rho}_B(t) \approx \hat{\rho}_B(\mathrm{T}) = \exp\left(-(1/k_\mathrm{B}T)\,\hat{H}_B\right)$. For the definition of the state of subsystem A the approximate disentanglement was postulated in the form of the relationship: $\hat{\rho}_{A+B}(t) \approx \hat{\rho}^d_{A+B(\mathrm{T})}(t) = \hat{\rho}_A(t) \otimes \hat{\rho}_B(\mathrm{T})$, known as the "first assumption of the quantum theory of relaxation" or the "basic condition of irreversibility" [8],[10],[11] (here and further by superscript d we note the approximate disentanglement).

Another approximate algorithm of disentanglement, widely utilized in the quantum informatics, is based on the positions of the quantum theory of measurements [12],[13],[14],[15], when both subsystems are relied by quantum. It was initially postulated that the indirect determination of the state of subsystem A is accomplished by means of the individual ideal projection quantum measurement on quasi-independent subsystem-pointer B [13],[15]. As a result of measurement of the observable $\hat{B}$ by external macroscopic gauge the subsystem B (according to the projection postulate of quantum mechanics) occurs in one of the eigenstates $|\beta\rangle$ of that observable. The density operator of subsystem-pointer after measurement is immediately considered equal: $\hat{\rho}_{B(\beta)}(t) = \hat{P}_B(\beta) = |\beta\rangle\langle\beta|$. The density operator of subsystem A in this case is defined as the result of the quantum averaging of the density operator $\hat{\rho}_{A+B}(t)$ over the subsystem B [15]:

$$\hat{\rho}_{A(\beta)}(t) = \frac{\mathrm{Sp}_B\left(\hat{\rho}_{A+B}(t)\hat{P}'_B(\beta)\right)}{\mathrm{Sp}_{AB}\left(\hat{\rho}_{A+B}(t)\hat{P}'_B(\beta)\right)}. \tag{9}$$

Disentangled state:
$$\hat{\rho}_{A+B}(t) \rightarrow \hat{\rho}^d_{A+B(\beta)} = \hat{\rho}_{A(\beta)}(t) \otimes \hat{P}_B(\beta), \tag{10}$$

is considered as the initial state (relative to the moment of measurement) in the description of the subsequent dynamics of composite quantum system, which leads to the quantum Zeno effect [15]. Such type indirect projection measurements (Zeno-like measurements) on the quasi-independent subsystem widely are discussed as one of the possible mechanisms of control of the state of subsystems in the quantum information theory.

In the recent works [16],[17] was shown the importance of the calculation of the entanglement of quantum subsystem with the external (with respect to the considered composite system) gauge in the description of the result of quantum measurement. Is noted, that by the consequence of this calculation can be the incomplete loss of quantum coherence subsystem, i.e., a difference in its local density operator after measurement from the linear combination of projectors.

In the enumerated non-Neumann's algorithms of the approximate disentanglement the desired state of the observed subsystem A is determined in the form of functional from the known density operator of the of composite system $\hat{\rho}_{A+B}(t)$ and density operator of

the use of a traditional Neumann's algorithm mathematically equivalent situation occurs - the state of composite system $\hat{\rho}_{A+B}(t)$ is known, and is postulated the algorithm of the calculation of the reduced density operator of subsystem A, $\hat{\rho}_{AN}(t)$. The inverse problem is obvious: to what state of subsystem B, $\hat{\rho}_{B?}(t)$ does correspond the Neumann's determination of the density operator of the "observable" subsystem A? Or, to what disentanglement $\hat{\rho}_{A+B}(t) \rightarrow \hat{\rho}_{AN} \otimes \hat{\rho}_{B?}(t)$ it does correspond? Its examination was the starting point of ours work.

2 Physical content of the operation of the reduction-disentanglement of von Neumann.

The necessary step of the approximate disentanglement is answer to the question: should be represented the density operator of composite system in the form of the tensor product of the local density operators of subsystems (tensor product structure [18]), or in the form (4) of the linear combination of such products? In the quantum information theory usually the first idea is postulated [12]. However, in the recent work [18] the arguments were formulated, which show that precisely this selection is dictated by the experimental determination of observables and interactions between the subsystems. The idea of work [18] lies in the fact that the determination of composite system includes the possibility of conducting of local operations and measurements. Such composite system from an experimental point of view appears as two quasi-independent correlated subsystems, to what its mathematical idea in the form of the tensor product of the local density operators of subsystems corresponds. Assuming the positions of work [18], we will represent the entangled density operator in the form of the tensor product of some correlated density operators of subsystems:

$$\hat{\rho}_{A+B}(t) \approx \hat{\rho}^d_{A+B)} = \hat{\rho}_{Ac}(t) \otimes \hat{\rho}_{Bc}(t),$$ (11)

where subscript c distinguishes the local correlated density operators from corresponding local reduced density operators.

We transform (11) by the method, analogous to the method, used by von Neumann for the proof of the mutual correlation of the average values of observables (see chapter 6.2 [19]). Let us multiply (11) to the right on $\hat{\rho}'_{Bc}(t)$ or $\hat{\rho}'_{Ac}(t)$ and let us take partial track on the subsystem B or A. Using the orthonormality of the correlated density operators of subsystems, we will obtain the system of two connected equations [20],[21],[22]:

$$\hat{\rho}_{Ac}(t) \approx \frac{\mathrm{Sp}_B\hat{\rho}_{A+B}(t)\hat{\rho}'_{Bc}(t)}{\mathrm{Sp}_{AB}\hat{\rho}_{A+B}(t)\hat{\rho}'_{Bc}(t)}, \quad \hat{\rho}_{Bc}(t) \approx \frac{\mathrm{Sp}_A\hat{\rho}_{A+B}(t)\hat{\rho}'_{Ac}(t)}{\mathrm{Sp}_{AB}\hat{\rho}_{A+B}(t)\hat{\rho}'_{Ac}(t)}.$$ (12)

Right sides of (12) are the normalized quantum averaging of the density operator of the closed system over one of subsystems. Each equation defines the density operator of one subsystem as functional from the density operator of the closed system and density operator of another subsystem. Actually, this simply the more convenient record of expression (11). Such relationships are not unique. Repeating the procedure of right multiplication it is possible to obtain the set of the expressions of the form:

$$\hat{\rho}_{Ac}(t) \approx \frac{\mathrm{Sp}_B\hat{\rho}_{A+B}(t)\,[\hat{\rho}'_{Bc}(t)]^m}{\mathrm{Sp}_{AB}\hat{\rho}_{A+B}(t)\,[\hat{\rho}'_{Bc}(t)]^m}, \quad \hat{\rho}_{Bc}(t) \approx \frac{\mathrm{Sp}_A\hat{\rho}_{A+B}(t)\,[\hat{\rho}'_{Ac}(t)]^m}{\mathrm{Sp}_{AB}\hat{\rho}_{A+B}(t)\,[\hat{\rho}'_{Ac}(t)]^m},$$ (13)

where m - arbitrary integer. It is obvious that essential it is possible to consider only

density operators of subsystems.

Now let us show, to what disentanglement does implicitly correspond von Neumann's reduction? Let us examine the first of expressions (13), which determine the state of the "observed" subsystem A. Assume that its correlated density operator is determined by von Neumann's reduction $\hat{\rho}_{Ac}(t) = \hat{\rho}_{AN}(t)$. Then precise equalities for all expressions (13) are carried out in two cases:

1. Or the density operator of subsystem B corresponds to the pure state, when $[\hat{\rho}_{Bc}(t)]^m = \hat{\rho}_{Bc}(t)$ (that corresponds physically to the limiting case of the noninteracting subsystems).

2. Or it is proportional to the unit operator $\hat{\rho}_{Bc}(t) = \hat{\rho}_{B\max}(t) = (1/N_B)\hat{I}_B$ (that corresponds to the steady state of subsystem B with the maximum entropy, the minimum information about subsystem or the infinite temperature according to [8]).

Since the trivial case of the noninteracting subsystems does not correspond to initial assumption about the entanglement of composite quantum system, remains the second version. Thus, the use of von Neumann's algorithm of the reduction for determining the local density operator of the observed subsystem is physically equivalent to the approximate disentanglement, with which the unobservable subsystem is relied by being stationarily been in the state with the maximum entropy or infinite temperature, in which it with the equal probability is found in any from the eigenstates. In this case quantum coherence (non-diagonal matrix elements of density matrix) in the subsystem B is equal to zero:

$$\hat{\rho}_{A+B}(t) \rightarrow \hat{\rho}_{A+B}^d(t) = \hat{\rho}_{AN}(t) \otimes \hat{\rho}_{B\max}. \tag{14}$$

To the same conclusion it is possible to come by another way, using positions of the quantum measurements theory [13],[14],[15]. Let the subsystem B be the quantum pointer, with the aid of which is accomplished the indirect measurement of the state of subsystem A. What must be the results of many projective measurements (1),(2) on the equally prepared system $\hat{\rho}_{A+B}(t)$, so that the result would be described by the density operator of $\hat{\rho}_{AN}(t)$?

Let $p(\beta)$ is the probability of observing the value β, which is appeared the eigenvalue of that observable $\hat{B}$, with the individual ideal projection quantum measurements on the quasi-independent subsystem B [15]. With each measurement the density operator of subsystem A is determined by the appropriate expression of form (2). Consequently, takeing into account the statistical nature of β measurements, the density operator of subsystem A it is necessary to define as the result of statistical averaging over the results of many individual projection measurements of $\hat{B}$:

$$\hat{\rho}_{AP(B)}(t) = \frac{\sum_\beta p(\beta)\mathrm{Sp}_B\left(\hat{\rho}_{A+B}\hat{P}'_B(\beta)\right)}{\mathrm{Sp}_A\sum_\beta p(\beta)\mathrm{Sp}_B\left(\hat{\rho}_{A+B}\hat{P}'_B(\beta)\right)}. \tag{15}$$

Using permutability of the operations of summation over β and the takings of the track on the subsystem B, let us write down the condition of the equality of this averaged density operator of subsystem A to the operator $\hat{\rho}_{AN}(t)$:

$$\frac{\mathrm{Sp}_B\left(\hat{\rho}_{A+B}(t)\left(\hat{I}_A \otimes \sum_\beta p(\beta)\hat{P}_B(\beta)\right)\right)}{} = \mathrm{Sp}\hat{\rho}_{A+B}(t). \tag{16}$$

Equality (16) is correct in the general case of arbitrary state only if $p(\beta) = (1/N_\beta)$. Thus, the description of the dynamics of the observed subsystem by the von Neumann's reduced density operator corresponds in the quantum measurements theory to the case, when quantum subsystem-pointer with the projection measurements with the equal probability is revealed in any of the eigenstates. The use of a projection postulate in the quantum measurements theory determines the total loss of quantum coherence by subsystem-pointer with each individual measurement [15],[2]. Comparing (16) and (9) we conclude that the state of subsystem-pointer in this case is physically equivalent to its presence in the state with the infinite temperature. By other words the use of von Neumann reduction with the disentanglement corresponds precisely to the "first assumption of the quantum relaxation theory" in the extreme case, when the temperature of thermostat is relied infinite [8],[10].

If we use an idea about the correlation of subsystems (12)-(13), then von Neumann's reduction [19], the "first assumption of the quantum theory of relaxation" [8],[10] and definition (9) [15] can be examined as special cases of approaching the assigned state of the "unobservable" subsystem in the disentanglement problem. In each case in the explicit or implicit form the state of one subsystem is postulated, and the state (density operator) of another subsystem correlated with it is determined by expressions (13). It is possible to conclude that in the conventional algorithm of the calculation of the average values of those observed by one of the subsystems the measuring projection postulate of quantum mechanics is implicitly used twice. For the first time with the determination of the density operator of subsystem from the known state of composite system (reduction-disentanglement of von Neumann), and for the second time with the calculation of the average values of the observables for the subsystem with the counted already known density operator. Our further consideration is based on the idea, that the first step (disentanglement) does not identify with the macroscopic projection measurement, and it must be based on the basis of the conditions of each specific objectives.

3   Correlated disentanglement.

The algorithms of the approximate disentanglement examined include in the explicit or implicit form assumption about state of one of the subsystems. By each concrete selection of the density operator of the "unobservable" subsystem is simulated the specific physical process of measurement and opposite effect of measurement on the state of subsystems. This idea for the projection measurements is contained in the quantum theory of measurements [13],[14],[15] in the form of the mathematical expressions of form (9), (15). For the clarity let us show differences in the calculated dynamics of the observed subsystem, caused by the selection of the operator of the density of the unobservable subsystem, based on the example to model in the quantum informats of the composite system of two qubits [12],[2].

Let us designate the eigenstates of the independent subsystems-qubits A and B, $|2\rangle_A, |1\rangle_A$ and $|2\rangle_B, |1\rangle_B$, respectively. The operators of composite system are represented by the matrices of the fourth order, whose each element is designated by two pairs of the subscripts $(\alpha\beta, \alpha'\beta')$ (see, for example, [10] Appendix A). The first index of each pair (α, α') corresponds to the eigenstate of subsystem A, and the second β, β' - subsystem B. In the general case the unitary dynamics of the closed system in question is described by

the density matrix of form [10]:

$$(\hat{\rho}_{A+B}(t)) = \begin{pmatrix} \rho_{22,22} & \rho_{22,21} & \rho_{22,12} & \rho_{22,11} \\ \rho_{21,22} & \rho_{21,21} & \rho_{21,12} & \rho_{21,11} \\ \rho_{12,22} & \rho_{12,21} & \rho_{12,12} & \rho_{12,11} \\ \rho_{11,22} & \rho_{11,21} & \rho_{11,12} & \rho_{11,11} \end{pmatrix}, \tag{17}$$

where dependence on the time is omitted for simplicity in the expressions of matrix elements.

According to (15) the dynamics of qubit A, determined from the results of many projective measurements of qubit B with the measured probabilities p_{B2}, p_{B1}, to reveal it in the states $|2\rangle_B, |1\rangle_B$, correspondingly, is described by the density operator:

$$\hat{\rho}_{AP(B)} = \frac{p_{B2}\mathrm{Sp}_B\left(\hat{\rho}_{A+B}\left(\hat{I}_A \otimes \hat{P}_{B2}\right)\right) + p_{B1}\mathrm{Sp}_B\left(\hat{\rho}_{A+B}\left(\hat{I}_A \otimes \hat{P}_{B1}\right)\right)}{\mathrm{Sp}_A\left(p_{B2}\mathrm{Sp}_B\left(\hat{\rho}_{A+B}\left(\hat{I}_A \otimes \hat{P}_{B2}\right)\right) + p_{B1}\mathrm{Sp}_B\left(\hat{\rho}_{A+B}\left(\hat{I}_A \otimes \hat{P}_{B1}\right)\right)\right)}. \tag{18}$$

Let $p_{B2} = p$, $p_{B1} = 1 - p$, i.e., the density matrix of the subsystem of qubit-pointer be relied by stationary and equal:

$$(\hat{\rho}_{B(P)}) = \begin{pmatrix} p & 0 \\ 0 & 1 - p \end{pmatrix}. \tag{19}$$

Then the local, correlated with it density matrix, which describes the dynamics of the observed subsystem A, is equal:

$$(\hat{\rho}_{AP(B)}) = \begin{pmatrix} p\rho_{22,22} + (1-p)\rho_{21,21} & p\rho_{22,12} + (1-p)\rho_{21,11} \\ p\rho_{12,22} + (1-p)\rho_{11,21} & p\rho_{12,12} + (1-p)\rho_{11,11} \end{pmatrix} \times$$
$$\times \left[p\rho_{22,22} + (1-p)\rho_{21,21} + p\rho_{12,12} + (1-p)\rho_{11,11}\right]^{-1}. \tag{20}$$

This result immediately is obtained, if we substitute (19) into the first of (12). Thus, the approximate disentanglement is realized in this case by the following replacement:

$$(\hat{\rho}_{A+B}) \rightarrow (\hat{\rho}_{A+B}^d) = (\hat{\rho}_{AP(B)}) \otimes (\hat{\rho}_{B(P)}) =$$
$$\begin{pmatrix} p^2\rho_{22,22} + p(1-p)\rho_{21,21} & 0 & p^2\rho_{22,12} + p(1-p)\rho_{21,11} & 0 \\ 0 & p(1-p)\rho_{22,22} + (1-p)^2\rho_{21,21} & 0 & p(1-p)\rho_{22,12} + (1-p)^2\rho_{21,11} \\ p^2\rho_{12,22} + p(1-p)\rho_{11,21} & 0 & p^2\rho_{12,12} + p(1-p)\rho_{11,11} & 0 \\ 0 & p(1-p)\rho_{12,22} + (1-p)^2\rho_{21,21} & 0 & p(1-p)\rho_{12,12} + (1-p)^2\rho_{11,11} \end{pmatrix} \times$$
$$\left[p\rho_{22,22} + (1-p)\rho_{21,21} + p\rho_{12,12} + (1-p)\rho_{11,11}\right]^{-1} \tag{21}$$

This nontraditional presentation of the operation of reduction, recorded in the "disentangled" form of the direct product of the local density operators of subsystems, makes it possible to clearly present the sense of the made approximations, comparing matrix elements (21) and (17). The first difference - the non-diagonal on the indices of subsystem-pointer matrix elements of the density matrix of composite system are assumed equal to zero. By this step is mathematically reflected the postulated loss by the subsystem-pointer of

on the subsystem [13],[2]. Let us note that in this case the quantum coherence in the subsystem A does not disappear, i.e., complete dekogerence of both the composite system and the observed subsystem does not occur. The second difference - the diagonal on the indices of subsystem-pointer elements of complete density matrix are substituted with their linear combinations with the weights, determined according to the results of local projection measurements on the subsystem-pointer B. With the equal probability of detecting qubit B in the eigenstates ($p = 1/2$) the approximate disentanglement corresponds to the von Neumann's reduction. With $p \neq 1/2$ joint use of both mathematical approximations is equivalent to physical assumption about the validity of the description of the state of the unobservable subsystem by the specific temperature T. It is evident that with $p \neq 1/2$ the matrix elements of the local density operator (20) quantitatively are differed from that obtained by traditional calculation and, correspondingly, the dynamics of the observed subsystem can differ from usually supposed.

Qualitative difference in the dynamics of the observed subsystem A will arise, if one assumes that the quantum coherence remains with the disentanglement in the subsystem B. To this assumption corresponds idea about that which the physical separation of composite system into the subsystems does not identify with conducting of projection macroscopic measurement. Either the physical separation of subsystems is accomplished before conducting of local projection measurement on qubit B, or the nondestructive quantum measurement is conducted . To the retention of quantum coherence in the subsystem B mathematically corresponds adding to the nondiagonal elements of the local density matrix $\hat{\rho}_B$ of values $|b| \neq 0$ instead of their equality to zero, postulated in (19):

$$(\hat{\rho}_{Bc}) = \begin{pmatrix} p & b \\ b^* & 1-p \end{pmatrix}, \quad (\hat{\rho}'_{Bc}) = \left(\hat{I}_A\right) \otimes (\hat{\rho}_{Bc}) = \begin{pmatrix} p & b & 0 & 0 \\ b^* & 1-p & 0 & 0 \\ 0 & 0 & p & b \\ 0 & 0 & b^* & 1-p \end{pmatrix}. \quad (22)$$

Using first of (12) we will obtain correlated to (22) the local density matrix of the subsystem A:

$$(\hat{\rho}_{Ac}) =$$
$$= \begin{pmatrix} p\rho_{22,22} + b^*\rho_{22,21} + & p\rho_{22,12} + b^*\rho_{22,11} + \\ +b\rho_{21,22} + (1-p)\rho_{21,21} & +b\rho_{21,12} + (1-p)\rho_{21,11} \\ p\rho_{12,22} + b^*\rho_{12,21} + & p\rho_{12,12} + b^*\rho_{12,11} + \\ +b\rho_{11,22} + (1-p)\rho_{11,21} & +b\rho_{11,12} + (1-p)\rho_{11,11} \end{pmatrix} \times$$
$$\times [p\left(\rho_{22,22} + \rho_{12,12}\right) + (1-p)((\rho_{21,21} + \rho_{11,11}) +$$
$$+ b\left(\rho_{21,22} + \rho_{11,12}\right) + b^*\left(\rho_{22,21} + \rho_{12,11}\right)]^{-1} \quad (23)$$

From comparison (23) with (20) it is evident that the dynamics of populations (diagonal elements $(\hat{\rho}_{Ac})$) and quantum coherence of qubit A, depends in this case not only on probabilities to reveal qubit B in the eigenstates, but also from its quantum coherence. The analogous (21) expression of the approximately disentangled density matrix of composite

system in this case is equal to:

$$(\hat{\rho}_{A+B}) \rightarrow \left(\hat{\rho}^d_{(A+B)c}\right) = \left(\hat{\rho}_{A(Bc)}\right) \otimes \left(\hat{\rho}_{Bc}\right) =$$

$$\left(
\begin{array}{cc}
\begin{array}{c} p^2\rho_{22,22} + p(1-p)\rho_{21,21}+ \\ pb^*\rho_{22,21} + pb\rho_{21,22} \end{array} & \begin{array}{c} pb\rho_{22,22} + (1-p)b\rho_{21,21}+ \\ |b|^2\rho_{22,21} + b^2\rho_{21,22} \end{array} \\
\begin{array}{c} pb^*\rho_{22,22} + (1-p)b^*\rho_{21,21}+ \\ (b^*)^2\rho_{22,21} + |b|^2\rho_{21,22} \end{array} & \begin{array}{c} p(1-p)\rho_{22,22} + (1-p)^2\rho_{21,21}+ \\ (1-p)b^*\rho_{22,21} + (1-p)b\rho_{21,22} \end{array} \\
\begin{array}{c} p^2\rho_{12,22} + p(1-p)\rho_{11,21}+ \\ pb^*\rho_{12,21} + pb\rho_{11,22} \end{array} & \begin{array}{c} pb\rho_{12,22} + (1-p)b\rho_{11,21}+ \\ |b|^2\rho_{12,21} + b^2\rho_{11,22} \end{array} \\
\begin{array}{c} pb^*\rho_{12,22} + (1-p)b^*\rho_{11,21}+ \\ (b^*)^2\rho_{12,21} + |b|^2\rho_{11,22} \end{array} & \begin{array}{c} p(1-p)\rho_{12,22} + (1-p)^2\rho_{11,21}+ \\ (1-p)b*\rho_{12,21} + (1-p)b\rho_{11,22} \end{array} \\
\begin{array}{c} p^2\rho_{22,12} + p(1-p)\rho_{21,11}+ \\ pb^*\rho_{22,11} + pb\rho_{21,12} \end{array} & \begin{array}{c} pb\rho_{22,12} + (1-p)b\rho_{21,11}+ \\ |b|^2\rho_{22,11} + b^2\rho_{21,12} \end{array} \\
\begin{array}{c} pb^*\rho_{22,12} + (1-p)b^*\rho_{21,11}+ \\ (b^*)^2\rho_{22,11} + |b|^2\rho_{21,12} \end{array} & \begin{array}{c} p(1-p)\rho_{22,12} + (1-p)^2\rho_{21,11}+ \\ (1-p)b^*\rho_{22,11} + (1-p)b\rho_{21,12} \end{array} \\
\begin{array}{c} p^2\rho_{12,12} + p(1-p)\rho_{11,11}+ \\ pb^*\rho_{12,11} + pb\rho_{11,12} \end{array} & \begin{array}{c} pb\rho_{12,12} + (1-p)b\rho_{11,11}+ \\ |b|^2\rho_{12,11} + b^2\rho_{11,22} \end{array} \\
\begin{array}{c} pb^*\rho_{12,12} + (1-p)b^*\rho_{11,11}+ \\ (b^*)^2\rho_{12,11} + |b|^2\rho_{11,12} \end{array} & \begin{array}{c} p(1-p)\rho_{12,12} + (1-p)^2\rho_{11,11}+ \\ (1-p)b*\rho_{12,11} + (1-p)b\rho_{11,12} \end{array}
\end{array}
\right) \quad (24)$$

$$\times \; [p\left(\rho_{22,22} + \rho_{12,12}\right) + (1-p)\left(\rho_{21,21} + \rho_{11,11}\right) + b\left(\rho_{21,22} + \rho_{11,12}\right) + b^*\left(\rho_{22,21} + \rho_{12,11}\right)]^{-1}.$$

From comparison (21),(24) and respectively (20),(23) it is evident that to idea about the retention of quantum coherence in the subsystem-pointer qualitatively corresponds the partial retention of the quantum coherence of composite system. In the general case all nondiagonal elements of density matrix (24) are not equal to zero. Simultaneously it is evident from this simplest example that the use of approximation of the assigned state of the subsystem-pointer (i.e. the selection of the model of measurement, including as a special case, the conventional algorithm of the reduction of von Neumann) leads always to redefining (change) of all matrix elements of the composite density matrix. Thus always with the realization of the mathematical operation of the approximate disentanglement occurs a change in the entropy or information about the composite quantum system [16] and partial decogerence [2].

Different dynamics of the observed subsystem is the consequence of different selection of the algorithm of disentanglement (model of measurement). The question about the criterion of the selection of the disentanglement algorithm arises and its correspondence to the experimental procedure of measurement accepted. Physically limiting cases are the von Neumann's disentanglement, which corresponds to complete decogerence of the subsystem-pointer and the "nondestructive" disentanglement, to which corresponds the invariability (in the limit) of the density operator of composite system. If the approximation of the closed system and nondestructive quantum measurement is correct, it is justified to search for the algorithm of separation, which minimizes change of $\hat{\rho}_{A+B}$ and simultaneously the change of entropy. Such local mutually correlated density operators of subsystems it is possible to obtain by solving system of equations (12) by the method

of sequential approximations [22],[21]:

$$\hat{\rho}_{Ac} = \lim_{n\to\infty} \hat{\rho}_A^{(n+1)} = \lim_{n\to\infty} \frac{\mathrm{Sp}_B \hat{\rho}_{A+B} \hat{\rho}_B'^{(n)}}{\mathrm{Sp}_{AB} \hat{\rho}_{A+B} \hat{\rho}_B'^{(n)}},$$

$$\hat{\rho}_{Bc} = \lim_{n\to\infty} \hat{\rho}_B^{(n+1)} = \lim_{n\to\infty} \frac{\mathrm{Sp}_A \hat{\rho}_{A+B} \hat{\rho}_A'^{(n)}}{\mathrm{Sp}_{AB} \hat{\rho}_{A+B} \hat{\rho}_B'^{(n)}}, \quad (25)$$

where, for example, the reduced density operators $\hat{\rho}_{AN}, \hat{\rho}_{BN}$ it is possible to use as zero approximation (n=0). One should emphasize that this correlated reduction corresponds to idea about the equivalence of the subsystems of the closed quantum system.

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